A Note on "The Cost of a Global Tariff War: A Sufficient Statistics Approach"*

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Abstract

This note derives exact optimal tariff formulas for the framework in Lashkaripour (2021), explicitly accounting for how extraterritorial relative wage and income adjustments affect Home's welfare. The exact characterization preserves the original sufficient-statistics structure: Nash tariffs can still be computed by solving a system of non-linear equations rather than a constrained optimization problem, delivering the same orders-of-magnitude gains in computational speed. Quantitatively, the resulting Nash tariffs and welfare estimates are virtually indistinguishable from Lashkaripour's (2021) baseline results. This outcome echoes the logic outlined in the original paper that unilateral tariffs elicit small extraterritorial wage and income adjustments, and that these adjustments furthermore constitute transfers within the rest of the world, leaving little effect on Home's welfare.

1 Introduction

In Lashkaripour (2020), I develop a simple sufficient statistics methodology to measure the prospective cost of a global tariff war in one simple step as a function of observable shares and industry-level trade elasticities. In the interest of exposition, I begin my analysis with a baseline multi-industry, multi-country Ricardian model that nests the Eaton and Kortum (2002) and Armington models as a special case. To derive analytic formulas for Nash tariffs that are compatible with my general equilibrium, multi-country and multi-industry analysis, the baseline derivation invokes two simple assumptions. First, treating the export partner's wage as the numeraire, the welfare effects of a tariff that occur through a change in relative wages in the rest of the world are assumed to be zero to a first-order approximation. Second, general equilibrium income effects in the

*Replication package: https://ahmadlp.github.io/global-tariff-war-replication.zip.

rest of the world—namely, the effects of wage adjustments on tariff revenues and total income in foreign destinations—are assumed to be zero to a first-order approximation.

In this note, I present the baseline Ricardian model using the same notation, explain the assumptions that implied the original optimal tariff formula, and derive an updated analytical characterization for unilaterally optimal tariffs without the above assumptions. I then show how to combine the updated optimal tariff formula with the equilibrium conditions using the exact hat-algebra methodology, popularized by Dekle et al. (2007), to compute the Nash tariffs and their welfare effects in one simple (optimization-free) step as a function of observable shares, applied tariffs, national income, and industry-level trade elasticities. Finally, I apply this methodology to actual trade and production data from 43 major economies and 56 industries over the 2000–2014 period to compare the Nash tariffs and their welfare effects under the original and updated formulas.

2 Theoretical Framework

Throughout my analysis, I consider a global economy consisting of $i = 1, \ldots, N$ countries and $k = 1, \ldots, K$ industries, with $\mathbb{C}$ and $\mathbb{K}$ respectively denoting the set of countries and industries. Labor is the only primary factor of production. Each country i is populated with $\bar{L}_i$ workers, each of whom supplies one unit of labor inelastically. Workers are perfectly mobile across industries but immobile across countries.

2.1 Demand

In the baseline Ricardian model, all varieties in industry k are differentiated by country of origin, with the triplet ji, k denoting a variety corresponding to origin j–destination i–industry k. Under the Eaton and Kortum (2002) interpretation of the model, national product differentiation of this kind can be attributed to Ricardian specialization within industries. The representative consumer in Country i maximizes a general utility function, which yields an indirect utility function as follows:

$$V_i(Y_i, \tilde{\mathbf{P}}_i) = \max_{\mathbf{Q}_i} U_i(\mathbf{Q}_i) \quad \text{s.t.} \quad \tilde{\mathbf{P}}_i \cdot \mathbf{Q}_i = Y_i. \tag{1}$$

In the above problem, Y_i denotes total income; $\mathbf{Q}_i = \{Q_{ji,k}\}$ denotes the vector of composite consumption quantities, $\tilde{\mathbf{P}}_i = \{\tilde{P}_{ji,k}\}$ denotes the corresponding vector of "consumer" price indexes, and "$\cdot$" is the inner product operator (i.e., $\mathbf{a} \cdot \mathbf{b} = \sum_i a_i b_i$). To avoid any confusion, I emphasize that tilde on the price variable is used to distinguish between (after-

tax) consumer and (pre-tax) producer prices. The representative consumer's problem yields a Marshallian demand function,

$$Q_{ji,k} = \mathcal{Q}_{ji,k}\left(Y_i, \tilde{\mathbf{P}}_i\right), \tag{2}$$

which describes optimal consumption in country i as a function of income, Y_i, and consumer prices, $\tilde{\mathbf{P}}_i$. When analyzing optimal tariff policy in each country, several demand-side variables play a key role. First, expenditure shares which represent the importance of each good in the consumption basket. Second, demand elasticities, which summarize the demand function specified under Equation 2. Below, I formally define these set of variables.

Definition 1 (Expenditure Shares)**.** *The share of country i's expenditure on industry k goods is denoted by $e_{i,k}$, and the within-industry share of expenditure on variety ji,k (origin j–destination i–industry k) is denoted by $\lambda_{ji,k}$:*

$$e_{i,k} \equiv \frac{\tilde{\mathbf{P}}_{i,k} \cdot \mathbf{Q}_{i,k}}{\tilde{\mathbf{P}}_i \cdot \mathbf{Q}_i} = \frac{\sum_{j=1}^{N} \tilde{P}_{ji,k} Q_{ji,k}}{Y_i}; \qquad \lambda_{ji,k} \equiv \frac{\tilde{P}_{ji,k} Q_{ji,k}}{\tilde{\mathbf{P}}_{i,k} \cdot \mathbf{Q}_{i,k}} = \frac{\tilde{P}_{ji,k} Q_{ji,k}}{e_{i,k} Y_i}.$$

Building on the above definitions, the unconditional expenditure share on variety ji,k ($e_{ji,k}$) and the overall share of expenditure on goods from origin j (λ_{ji}) is defined as

$$e_{ji,k} \equiv \lambda_{ji,k} e_{i,k}; \qquad \lambda_{ji} \equiv \sum_{k=1}^{K} \lambda_{ji,k} e_{i,k}.$$

Note the distinction between $e_{ji,k}$ and $\lambda_{ji,k}$. The former concerns the share of variety ji,k in total expenditure. The latter concerns the share of expenditure on variety ji,k conditional on buying industry k goods. As we will see shortly, $\lambda_{ji,k}$ governs the Marshallian demand elasticities under CES preferences. These elasticities are defined as follows for the general (not-necessarily CES) case.

Definition 2 (Demand Elasticities)**.** *The elasticity of demand for good ji,k with respect to the price of good ni,g is denoted by*

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \partial \ln \mathcal{Q}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i) / \partial \ln \tilde{P}_{ni,g}. \tag{3}$$

Correspondingly, the matrix of "nominal" and "expenditure-adjusted" demand elasticities are

denoted by

$$\mathbf{E}_{ji}^{(ni)} \equiv \begin{bmatrix} \varepsilon_{ji,1}^{(ni,1)} & \cdots & \varepsilon_{ji,1}^{(ni,K)} \\ \vdots & \ddots & \vdots \\ \varepsilon_{ji,K}^{(ni,1)} & \cdots & \varepsilon_{ji,K}^{(ni,K)} \end{bmatrix}; \qquad \tilde{\mathbf{E}}_{ji}^{(ni)} \equiv \begin{bmatrix} e_{ji,1}\varepsilon_{ji,1}^{(ni,1)} & \cdots & e_{ji,1}\varepsilon_{ji,1}^{(ni,K)} \\ \vdots & \ddots & \vdots \\ e_{ji,K}\varepsilon_{ji,K}^{(ni,1)} & \cdots & e_{ji,K}\varepsilon_{ji,K}^{(ni,K)} \end{bmatrix},$$

with $\mathbf{E}_{ji} \sim \mathbf{E}_{ji}^{(ji)}$ *denoting the matrix of own-price elasticities of demand.*

I assume that consumer preferences are well-behaved in that $\varepsilon_{ji,k}^{(ji,k)} < -1$. We can appeal to two properties of the Marshallian demand function, namely, (i) Cournot aggregation, and (ii) homogeneity of degree zero, to prove that the elasticity matrixes, $\mathbf{E}_{ji}$ and $\tilde{\mathbf{E}}_{ji}$, are invertible.

Lemma 1. *The matrixes* $\mathbf{E}_{ji} \sim \mathbf{E}_{ji}^{(ji)}$ *and* $\tilde{\mathbf{E}}_{ji} \sim \tilde{\mathbf{E}}_{ji}^{(ji)}$ *are non-singular.*

One commonly-used specification in the quantitative trade literature is the Cobb-Douglas-CES specification. Namely,

$$U_i(\mathbf{Q}_i) = \prod_{k=1}^{K} \left(\sum_{j=1}^{N} \bar{\varsigma}_{ji,k} Q_{ji,k}^{\rho_k} \right)^{e_{i,k}/\rho_k}, \qquad \lambda_{ji,k} = \frac{\bar{\varsigma}_{ji,k} \tilde{P}_{ji,k}^{-\epsilon_k}}{\sum_{n=1}^{N} \left(\bar{\varsigma}_{ni,k} \tilde{P}_{ni,k}^{-\epsilon_k} \right)}, \tag{4}$$

where $\bar{\varsigma}_{ji,k}$ is a structural demand shifter and $\epsilon_k \equiv \rho_k/(1-\rho_k)$ denotes the industry-level trade elasticity. Under this specification, the cross-price elasticities of demand between varieties from different industries collapse to zero, while the remaining elasticities are fully characterized by $\lambda_{ji,k}$'s and ϵ_k's:

$$\varepsilon_{ij,k}^{(ij,k)} = -1 - \epsilon_k(1 - \lambda_{ij,k}); \qquad \varepsilon_{nj,k}^{(ij,k)} = \epsilon_k \lambda_{ij,k}; \qquad \varepsilon_{ij,k}^{(ij,g)} = 0. \tag{5}$$

Under the Cobb-Douglas-CES utility, social welfare in Country i can be expressed as $W_i \equiv V_i(Y_i, \tilde{\mathbf{P}}_i) = Y_i/\tilde{P}_i$, where $\tilde{P}_i = \prod_{k=1}^{K} \tilde{P}_{i,k}^{e_{i,k}}$ and $\tilde{P}_{i,k} = \left(\sum_{j=1}^{N} \bar{\varsigma}_{ji,k} \tilde{P}_{ji,k}^{-\epsilon_k} \right)^{-1/\epsilon_k}$ is the aggregate consumer price index.

2.2 Production

In the baseline Ricardian model, labor is the sole factor of production and the unit labor cost of production and transportation is invariant to policy. Correspondingly, the "producer" price of composite variety ji,k can be expressed as a function of the labor

wage rate in country j, w_j, multiplied by the constant unit labor cost of production, $\bar{a}_{j,k}$, and the iceberg trade cost, $\bar{\tau}_{ji,k}$ (with $\bar{\tau}_{ii,k} = 1$), and the "consumer" price equals the "producer" price times the tariff applied by country i on variety ji,k, namely, $t_{ji,k}$ (with $t_{ii,k} = 0$):

$$P_{ji,k} = \bar{\tau}_{ji,k}\bar{a}_{j,k}w_j, \qquad \tilde{P}_{ji,k} = (1 + t_{ji,k})P_{ji,k}. \tag{6}$$

The bar notation indicates that $\bar{a}_{j,k}$ and $\bar{\tau}_{ji,k}$ are invariant to policy. The invariance of $\bar{a}_{j,k}$ to policy change derives from constant returns to scale technologies. It amounts to a flat export supply curve, which entails that the passthrough of taxes onto consumer prices is complete after we net out general equilibrium wage effects.

2.3 General Equilibrium

Given the vector of tariffs in each country i, $\mathbf{t}_i = \{t_{ji,k}\}$, equilibrium consists of a vector of wages, $\mathbf{w} = \{w_j\}$, a vector of "producer" and "consumer" price indexes, $\mathbf{P}_i = \{P_{ji,k}\}$ and $\tilde{\mathbf{P}}_i = \{\tilde{P}_{ji,k}\}$ (as described by Equation 6), and consumption quantities, $\mathbf{Q}_i$, given by the Marshallian demand function 2, such that wage income in each country equals sales net of taxes,

$$w_i\bar{L}_i = \sum_{j=1}^{N}\sum_{k=1}^{K}\left[P_{ij,k}Q_{ij,k}\right] = \sum_{j=1}^{N}\sum_{k=1}^{K}\left[\frac{1}{1+t_{ij,k}}\lambda_{ij,k}e_{j,k}Y_j\right], \tag{7}$$

and total income equals the wage bill plus tariff revenue:

$$Y_i = w_i\bar{L}_i + \sum_{j\neq i}\sum_{k=1}^{K}\left[t_{ji,k}P_{ji,k}Q_{ji,k}\right] = w_i\bar{L}_i + \sum_{j\neq i}\sum_{k=1}^{K}\left[\frac{t_{ji,k}}{1+t_{ji,k}}\lambda_{ji,k}e_{i,k}Y_i\right]. \tag{8}$$

The above equation along with the representative consumer's budget constraint, ensure that trade is balanced between countries. For the reader's convenience, Table 1 reports a summary of the key variables and parameters of the model.

Table 1: Summary of Key Variables

Variable	Description
$\tilde{P}_{ji,k}$	Consumer price index of variety ji,k (origin j–destination i–industry k)
$P_{ji,k}$	Producer price index of variety ji,k (origin j–destination i–industry k)
$Q_{ji,k}$	Consumption quantity/Output of variety ji,k
$\chi_{ji,k}$	Share of variety ji,k in origin j's total exports ($j \neq i$)
Y_i	Total income in country i
$w_i\bar{L}_i$	Wage income in country i (wage $\times$ population size)
$t^*_{ji,k}$	Nash/Optimal tariff imposed by country i on variety ji,k
$\bar{t}_{ji,k}$	Applied (status-quo) tariff on variety ji,k
$e_{i,k}$	Country i's expenditure share on industry k
$\lambda_{ji,k}$	Expenditure share on variety ji,k: $\lambda_{ji,k} = \tilde{P}_{ji,k}Q_{ji,k}/e_{i,k}Y_i$
$r_{ji,k}$	Revenue share from variety ji,k: $r_{ji,k} = P_{ji,k}Q_{ji,k}/w_j\bar{L}_j$
$\varepsilon^{(ni,g)}_{ji,k}$	Price elasticity of demand: $\varepsilon^{(ni,g)}_{ji,k} = \partial \ln \mathcal{Q}_{ji,k}/\partial \ln \tilde{P}_{ni,g}$
ϵ_k	Constant trade elasticity under CES preferences

3 Assumptions in Lashkaripour (2020)

Social Welfare and the Optimal Policy Problem. Provided that equilibrium is unique, all equilibrium variables can be uniquely characterized as a function of global tariff rates, $\mathbf{t}$, and wages, $\mathbf{w}$, with the latter implicitly depending on tariffs, i.e., $\mathbf{w} = \mathbf{w}(\mathbf{t})$. Social welfare in Country i can, accordingly, be expressed as follows given the indirect utility function:

$$W_i(\mathbf{t}_i, \mathbf{t}_{-i}; \mathbf{w}) \equiv V_i\left(Y_i(\mathbf{t}_i, \mathbf{t}_{-i}; \mathbf{w}), \tilde{\mathbf{P}}_i(\mathbf{t}_i, \mathbf{t}_{-i}; \mathbf{w})\right).$$

In the event of a global tariff war, each country i sets their vector of unilaterally optimal tariffs $\mathbf{t}^*_i$, given applied tariffs in the rest of the world, $\mathbf{t}_{-i}$. The unilaterally optimal tariff, $\mathbf{t}^*_i = \mathbf{t}^*_i(\mathbf{t}_{-i})$, which describes country i's best non-cooperative response to $\mathbf{t}_{-i}$, solves the following problem:

$$\mathbf{t}^*_i(\mathbf{t}_{-i}) = \arg\max_{\mathbf{t}_i} \; W_i(\mathbf{t}_i; \mathbf{t}_{-i}; \mathbf{w}), \tag{P1}$$

where recall that the wage vector, $\mathbf{w} = \mathbf{w}(\mathbf{t}_i; \mathbf{t}_{-i})$, is itself an implicit function of applied tariffs all over the world. Considering the above problem, we can define the non-cooperative Nash equilibrium that transpires in the event of a global tariff war as follows.

Definition 3 (The Non-Cooperative Nash Equilibrium)**.** *A global tariff war corresponds to a non-cooperative Nash equilibrium in which all countries simultaneously set their vector of optimal*

tariffs, taking applied tariffs by the rest of the world as given. The Nash tariffs, therefore, solve the following system:

$$\mathbf{t}_i = \mathbf{t}_i^*(\mathbf{t}_{-i}), \qquad i = 1, \ldots, N,$$

where $\mathbf{t}_i^*(\mathbf{t}_{-i})$ is the unilaterally optimal tariff response implied by Problem (P1).

Treating tariffs in the rest of world as given (i.e., $\mathbf{t}_{-i} = \bar{\mathbf{t}}_{-i}$), country i's marginal welfare gain from imposing $t_{ji,k}$ can be calculated as

$$\frac{\mathrm{d}W_i(\mathbf{t}_i, \bar{\mathbf{t}}_{-i}; \mathbf{w})}{\mathrm{d}\ln(1+t_{ji,k})} = \frac{\partial W_i(\mathbf{t}_i, \bar{\mathbf{t}}_{-i}; \mathbf{w})}{\partial \ln(1+t_{ji,k})} + \left(\frac{\partial W_i(\mathbf{t}_i, \bar{\mathbf{t}}_{-i}; \mathbf{w})}{\partial \ln \mathbf{w}}\right)_{\mathbf{t}} \cdot \frac{\mathrm{d}\ln \mathbf{w}}{\mathrm{d}\ln(1+t_{ji,k})}. \tag{9}$$

The first term in the above equation accounts for the direct effect of tariffs on consumer prices and tariff revenues, holding $\mathbf{w}$ fixed. The second term accounts for the welfare effects that are mediated through general equilibrium wage adjustments. $\mathrm{d}\ln \mathbf{w}/\mathrm{d}\ln(1+t_{ji,k})$ can be calculated by applying the Implicit Function Theorem to the system of national labor market clearing conditions (Equation 7). To derive sufficient statistics formulas for Nash tariffs in Lashkaripour (2020), I invoke two assumptions regarding general equilibrium adjustments in the rest of the world.

Assumption 1: Relative Wage Effects in the Rest of the World. Assign wage in country j as the numeraire: $w_j = 1$. The last term in Equation 9 can be decomposed as

$$\left(\frac{\partial W_i}{\partial \ln \mathbf{w}}\right)_{\mathbf{t}} \cdot \frac{\mathrm{d}\ln \mathbf{w}}{\mathrm{d}\ln(1+t_{ji,k})} = \left(\frac{\partial W_i}{\partial \ln w_i}\right)_{\mathbf{t},\mathbf{w}_{-i}} \frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln(1+t_{ji,k})} + \left(\frac{\partial W_i}{\partial \ln \mathbf{w}_{-i}}\right)_{\mathbf{t},w_i} \cdot \frac{\mathrm{d}\ln \mathbf{w}_{-i}}{\mathrm{d}\ln(1+t_{ji,k})}.$$

Let $r_{ni,k} \equiv P_{ni,k}Q_{ni,k}/w_n\bar{L}_n$ denote the revenue share from variety ni,k, and let $r_{ni} \equiv \mathbf{P}_{ni} \cdot \mathbf{Q}_{ni}/w_n\bar{L}_n = \sum_{k=1}^{K} r_{ni,k}$ denote the share of origin n's wage revenue from sales to destination i. It is straightforward to cross-check from actual trade data that $r_{ni}/r_{ii} \approx 0$ if $n \neq i$. Stated verbally, each individual foreign destination accounts for a negligible fraction of country i's national income. Based on this observation and assigning w_j as the numeraire, the change in country i's welfare can be approximated as

$$\frac{\mathrm{d}W_i(\mathbf{t}_i, \bar{\mathbf{t}}_{-i}; \mathbf{w})}{\mathrm{d}\ln(1+t_{ji,k})} \approx \frac{\partial W_i(\mathbf{t}_i, \bar{\mathbf{t}}_{-i}; \mathbf{w})}{\partial \ln(1+t_{ji,k})} + \left(\frac{\partial W_i(\mathbf{t}_i, \bar{\mathbf{t}}_{-i}; \mathbf{w})}{\partial \ln w_i}\right)_{\mathbf{t},\mathbf{w}_{-i}} \frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln(1+t_{ji,k})}. \tag{10}$$

The above approximation posits that $t_{ji,k}$ can affect W_i by raising w_i relative to wages in the rest of world, $\mathbf{w}_{-i}$. But treating w_j as the numeraire, the welfare effects of $t_{ji,k}$ that occur through a change in w_n/w_j are zero to a first-order approximation iff $n \neq i$ and j. So, by

choice of numeraire, $\mathbf{w}_{-i} = \bar{\mathbf{w}}_{-i}$ is treated as fixed when calculating $\mathrm{d}\ln w_i / \mathrm{d}\ln(1+t_{ji,k})$.

Assumption 2: Income Effects in the Rest of the World. Using the above approximation, Proposition 1 of Lashkaripour (2020) states that country i's optimal non-cooperative import tariff is uniform across all exporters $j \neq i$ and industries k, and under the Cobb-Douglas-CES specification (Equations 4–5) is characterized by the following formula:

$$t_i^*(\mathbf{t}_{-i}) = \frac{1}{\sum_{k=1}^{K}\sum_{j\neq i}\left(\chi_{ij,k}^{*}\epsilon_k\left[1-(1-\delta_{j,k}^{*})\lambda_{ij,k}^{*}\right]\right)}, \tag{11}$$

where $\chi_{ij,k} \equiv \frac{P_{ij,k}Q_{ij,k}}{\sum_{n\neq i}\mathbf{P}_{in}\cdot\mathbf{Q}_{in}}$ is the share of variety ij,k in origin i's total exports, and $\delta_{j,k} \equiv \frac{t_j\lambda_{jj,k}e_{j,k}}{1+t_j\lambda_{jj}}$ accounts for the general equilibrium effect of country i's tariff on country j's tariff revenue under a uniform foreign tariff t_j while treating $\mathbf{w}_{-i} = \bar{\mathbf{w}}_{-i}$ as fixed. In the quantitative implementation of the baseline model in Lashkaripour (2020), $\delta_{j,k}^{*}$ is set to zero, which overlooks the general equilibrium effect of tariffs on tariff revenues and income in destination j.

Furthermore, Appendix D of Lashkaripour (2020) derives sufficient statistics formulas for Nash tariffs without the relative-wage approximation in Equation 10 by applying the Implicit Function Theorem to the system of national labor market clearing conditions:

$$F_n(\tilde{\mathbf{P}}_i, \mathbf{t}_{-i}; \mathbf{w}) \equiv w_n\bar{L}_n - \sum_{\ell=1}^{N}\left[\mathbf{P}_{n\ell}(w_n)\cdot\mathbf{Q}_{n\ell}(\tilde{\mathbf{P}}_i, \mathbf{t}_{-i}; \mathbf{w})\right] = 0, \qquad n = 1,\ldots,N.$$

However, when taking partial derivatives from the above system with respect to $\mathbf{w}$ under Cobb-Douglas-CES preferences, Appendix B and Appendix D specify the matrix $(\partial\mathbf{F}/\partial\ln\mathbf{w})_{\tilde{\mathbf{P}}_i,\mathbf{t}_{-i}} = \mathbf{I} - \mathbf{\Lambda}$ using elements $\Psi_{n\ell} \equiv \sum_{k=1}^{K} r_{n\ell,k}\left(1+\epsilon_k(\lambda_{\ell\ell,k} - \mathbf{1}_{n=\ell})\right)$, treating total income Y_ℓ in every foreign destination $\ell \neq i$ as proportional to wage income $w_\ell\bar{L}_\ell$, so that $\partial\ln Y_\ell/\partial\ln w_a = \mathbf{1}_{a=\ell}$. By Equation 8, however, total income in destination ℓ equals the wage bill plus tariff revenue:

$$Y_\ell = \frac{w_\ell\bar{L}_\ell}{\rho_\ell}, \qquad \rho_\ell \equiv \frac{w_\ell\bar{L}_\ell}{Y_\ell} = 1 - \sum_{n\neq\ell}\sum_{k=1}^{K}\frac{t_{n\ell,k}}{1+t_{n\ell,k}}\lambda_{n\ell,k}e_{\ell,k} = \sum_{n=1}^{N}\sum_{k=1}^{K}\frac{\lambda_{n\ell,k}e_{\ell,k}}{1+t_{n\ell,k}}. \tag{12}$$

When wages $\mathbf{w}$ change in general equilibrium (holding applied tariffs $\bar{\mathbf{t}}_{-i}$ in the rest of the world fixed), expenditure shares $\lambda_{n\ell,k}$ in foreign destinations $\ell \neq i$ change, which changes ρ_ℓ, foreign tariff revenue, and foreign total income Y_ℓ. Treating ρ_ℓ as invariant to $\mathbf{w}$ (or setting $\delta_{j,k}^{*} = 0$) assumes that income effects in the rest of the world have no

first-order effect on country i's welfare. Moreover, when foreign tariffs $\bar{t}_{n\ell,k}$ are non-zero and discriminatory, the zero-tariff identity $P_{j\ell,k}Q_{j\ell,k}\lambda_{a\ell,k} = P_{a\ell,k}Q_{a\ell,k}\lambda_{j\ell,k}$ does not hold; instead,

$$X_{j\ell,k}\lambda_{a\ell,k} = \frac{1+\bar{t}_{a\ell,k}}{1+\bar{t}_{j\ell,k}} X_{a\ell,k}\lambda_{j\ell,k}, \qquad \ell \neq i, \tag{13}$$

where $X_{n\ell,k} \equiv P_{n\ell,k}Q_{n\ell,k} = \frac{\lambda_{n\ell,k}e_{\ell,k}Y_\ell}{1+t_{n\ell,k}}$ denotes sales net of taxes from origin n to destination ℓ in industry k.

4 Optimal Tariffs Without Approximation

Below, I derive the exact unilaterally optimal tariff formula for country i in the baseline Ricardian model under Cobb-Douglas-CES preferences (Equation 4), without the approximation that relative wage changes in the rest of the world or income effects in the rest of the world have zero first-order effect on country i's welfare. Throughout this derivation, country i chooses all $(N-1)K$ import tariffs $\mathbf{t}_i = \{t_{ni,k}\}_{n\neq i,k}$ (with $1+t_{ni,k} > 0$ and $t_{ii,k} = 0$), taking applied tariffs in the rest of the world, $\bar{t}_{nd,k}$ (with $1+\bar{t}_{nd,k} > 0$ and $\bar{t}_{dd,k} = 0$ for all $d \neq i$), as given.

Let $I_n \equiv w_n \bar{L}_n$ denote wage income in country n, and let $X_{nd,k} \equiv P_{nd,k}Q_{nd,k} = \frac{\lambda_{nd,k}e_{d,k}Y_d}{1+t_{nd,k}}$ denote sales net of taxes from origin n to destination d in industry k. From Equation 12, the ratio of wage income to total income in destination d, $\rho_d \equiv I_d/Y_d$, satisfies

$$\rho_d = \sum_{n=1}^{N}\sum_{k=1}^{K} \frac{\lambda_{nd,k}e_{d,k}}{1+t_{nd,k}}, \qquad Y_d = \frac{I_d}{\rho_d}, \qquad \sum_{n=1}^{N}\sum_{k=1}^{K} X_{nd,k} = I_d. \tag{14}$$

Labor market clearing (Equation 7) requires $I_n = \sum_{d=1}^{N}\sum_{k=1}^{K} X_{nd,k}$ for $n = 1, \ldots, N$. Keep the expenditure, wage-revenue, and national-income shares:

$$\lambda_{nd} = \sum_{k=1}^{K} \lambda_{nd,k}e_{d,k}, \qquad r_{nd,k} = \frac{X_{nd,k}}{I_n}, \qquad r_{nd} = \sum_{k=1}^{K} r_{nd,k}, \qquad y_n = \frac{Y_n}{\sum_{a=1}^{N} Y_a}. \tag{15}$$

Labor market clearing implies $\sum_{d=1}^{N} r_{nd} = 1$. Since domestic sales are not subject to tariffs ($t_{nn,k} = 0$), we have $r_{nn} = \frac{\lambda_{nn}Y_n}{I_n} = \frac{\lambda_{nn}}{\rho_n}$. Accordingly, the following variables can be inferred directly from observable shares:

$$\rho_n = \frac{\lambda_{nn}}{r_{nn}}, \qquad \omega_n \equiv \rho_n y_n, \qquad \frac{\omega_n}{\omega_d} = \frac{I_n}{I_d}. \tag{16}$$

Next, define the destination wage-income shares:

$$q_{nd,k} \equiv \frac{\omega_n}{\omega_d} r_{nd,k} = \frac{X_{nd,k}}{I_d}, \qquad q_{nd} = \sum_{k=1}^{K} q_{nd,k}, \qquad Q_{d,k} = \sum_{a=1}^{N} q_{ad,k}, \tag{17}$$

which satisfy $\sum_{n=1}^{N} q_{nd} = \sum_{k=1}^{K} Q_{d,k} = 1$. For any origin n and any foreign destination $d \neq i$, define

$$\zeta_{nd} \equiv \sum_{k=1}^{K} \epsilon_k \left(Q_{d,k} \lambda_{nd,k} - q_{nd,k} \right). \tag{18}$$

As shown in Appendix A, ζ_{nd} equals the elasticity of ρ_d with respect to origin n's wage w_n, holding applied tariffs in the rest of the world fixed. It therefore accounts for the general equilibrium effect of wage adjustments on tariff revenue and total income in foreign destination d.

For foreign exporters $n, j \neq i$, define

$$C_{nj} \equiv \frac{\omega_j}{\omega_n} \left[r_{jn} + \sum_{d \neq i} \sum_{k=1}^{K} \epsilon_k r_{jd,k} \lambda_{nd,k} - \sum_{d \neq i} r_{jd} \zeta_{nd} \right], \tag{19}$$

$$D_n \equiv \sum_{d \notin \{i,n\}} r_{nd} + \sum_{d \neq i} \sum_{k=1}^{K} \epsilon_k r_{nd,k} (1 - \lambda_{nd,k}) + \sum_{d \neq i} r_{nd} \zeta_{nd}. \tag{20}$$

All terms in Equations 19 and 20 are characterized as a function of observable shares $\lambda_{nd,k}$, λ_{nd}, $r_{nd,k}$, r_{nd}, and y_n, and industry-level trade elasticities ϵ_k. Noting these definitions, we can characterize country i's unilaterally optimal tariff schedule as follows.

Proposition 1. *If preferences are described by functional form 4, country i's optimal non-cooperative import tariff is uniform across industries within each exporter $n \neq i$:*

$$t^*_{ni,k} = t^*_{ni,g} \equiv t^*_{ni}, \qquad \forall n \neq i,\ k, g \in \mathbb{K}. \tag{21}$$

For each exporter $n \neq i$, the optimal tariff is characterized by the following formula:

$$1 + t^*_{ni} = 1 + \frac{r^*_{ni} + \sum_{j \notin \{i,n\}} \left(\dfrac{\lambda^*_{ji} y^*_i}{\rho^*_j r^*_{ji} y^*_j} - 1 \right) C^*_{nj}}{D^*_n}, \tag{22}$$

as a function of observable shares and trade elasticities in the counterfactual equilibrium (denoted by $*$), where ζ_{nd}, C_{nj}, and D_n are given by Equations 18–20, $\rho_j = \lambda_{jj}/r_{jj}$, and the denominator D^*_n is strictly positive.

Special Cases. Before moving forward, it is useful to discuss how Equation 22 (the proof of which is provided in Appendix A) relates to special cases. First, if applied tariffs in the rest of the world are zero ($\bar{t}_{nd,k} = 0$ for all $d \neq i$), then $\rho_d = 1$, $q_{nd,k} = e_{d,k}\lambda_{nd,k}$, $Q_{d,k} = e_{d,k}$, and $\zeta_{nd} = 0$ for all $d \neq i$. By Equation 13, C_{nj} reduces to $\lambda_{jn} + \sum_{d\neq i}\sum_{k=1}^{K} \epsilon_k r_{nd,k}\lambda_{jd,k}$, and Equation 22 collapses to

$$1 + t^*_{ni} = 1 + \frac{r^*_{ni} + \sum\limits_{j\notin\{i,n\}} \left(\frac{\lambda^*_{ji} y^*_i}{r^*_{ji} y^*_j} - 1\right)\left[\lambda^*_{jn} + \sum\limits_{d\neq i}\sum\limits_{k=1}^{K} \epsilon_k r^*_{nd,k}\lambda^*_{jd,k}\right]}{\sum\limits_{d\notin\{i,n\}} r^*_{nd} + \sum\limits_{d\neq i}\sum\limits_{k=1}^{K} \epsilon_k r^*_{nd,k}(1-\lambda^*_{nd,k})}.$$

Second, in a two-country setup ($N = 2$), there is only one foreign exporter $n \neq i$, so the sum over $j \notin \{i, n\}$ is zero and $t^*_{ni} = r^*_{ni}/D^*_n > 0$. Third, in a single-industry setup ($K = 1$), $Q_{d,1} = 1$ and $\zeta_{nd} = \epsilon_1(\lambda_{nd} - q_{nd})$, which yields $C_{nj} = q_{jn} + \epsilon_1 \sum_{d\neq i} r_{nd} q_{jd}$ and $D_n = \sum_{d\notin\{i,n\}} r_{nd} + \epsilon_1 \sum_{d\neq i} r_{nd}(1 - q_{nd})$.

5 Computing Nash Tariffs Using Proposition 1

We can employ Proposition 1 to measure the prospective cost of a global tariff war without performing the iterative numerical optimization procedure. To compute the Nash equilibrium, we can employ the hat-algebra notation, whereby $\hat{x} \equiv x^*/x$ denotes the change in variable x when tariffs are elevated from their applied rate, $\bar{t}_{ji,k}$, to the Nash rate, t^*_{ji}.

Observing that by definition $\lambda^*_{ji,k} = \hat{\lambda}_{ji,k}\lambda_{ji,k}$, capitalizing on the multiplicatively-separable structure of the CES demand system allows us to express $\hat{\lambda}_{ji,k}$ as follows:

$$\hat{\lambda}_{ji,k} = \frac{\left[(1+\widehat{t}_{ji,k})\hat{w}_j\right]^{-\epsilon_k}}{\sum_{n=1}^{N}\left(\lambda_{ni,k}\left[(1+\widehat{t}_{ni,k})\hat{w}_n\right]^{-\epsilon_k}\right)} = \frac{\left[\left(\frac{1+t^*_{ji}}{1+\bar{t}_{ji,k}}\right)\hat{w}_j\right]^{-\epsilon_k}}{\sum_{n=1}^{N}\left(\lambda_{ni,k}\left[\left(\frac{1+t^*_{ni}}{1+\bar{t}_{ni,k}}\right)\hat{w}_n\right]^{-\epsilon_k}\right)}, \tag{23}$$

where $\bar{t}_{ji,k}$ denotes the applied (status-quo) tariff on good ji,k, and $t^*_{ii} = \bar{t}_{ii,k} = 0$. Correspondingly, counterfactual sales net of taxes from origin n to destination d in industry k are given by

$$X^*_{nd,k} = \frac{1}{1+t^*_{nd}}\hat{\lambda}_{nd,k}\lambda_{nd,k}e_{d,k}\hat{Y}_d Y_d, \qquad X^*_{nd} = \sum_{k=1}^{K} X^*_{nd,k}, \tag{24}$$

and counterfactual wage income is $I_n^* = \hat{w}_n w_n \bar{L}_n$. Using the counterfactual sales flows $X_{nd,k}^*$ and expenditure shares $\lambda_{nd,k}^* = \hat{\lambda}_{nd,k}\lambda_{nd,k}$, the counterfactual shares and terms in Proposition 1 for each tariff-imposing country i and foreign exporter $n \neq i$ are given by

$$r_{nd,k}^* = \frac{X_{nd,k}^*}{\hat{w}_n w_n \bar{L}_n}, \qquad r_{nd}^* = \sum_{k=1}^{K} r_{nd,k}^*, \qquad q_{nd,k}^* = \frac{X_{nd,k}^*}{\hat{w}_d w_d \bar{L}_d}, \qquad Q_{d,k}^* = \sum_{a=1}^{N} q_{ad,k}^*, \tag{25}$$

$$\zeta_{nd}^* = \sum_{k=1}^{K} \epsilon_k \left(Q_{d,k}^* \hat{\lambda}_{nd,k}\lambda_{nd,k} - q_{nd,k}^* \right), \tag{26}$$

$$C_{nj}^{i*} = \frac{\hat{w}_j w_j \bar{L}_j}{\hat{w}_n w_n \bar{L}_n} \left[r_{jn}^* + \sum_{d \neq i} \sum_{k=1}^{K} \epsilon_k r_{jd,k}^* \hat{\lambda}_{nd,k}\lambda_{nd,k} - \sum_{d \neq i} r_{jd}^* \zeta_{nd}^* \right], \tag{27}$$

$$D_n^{i*} = \sum_{d \notin \{i,n\}} r_{nd}^* + \sum_{d \neq i} \sum_{k=1}^{K} \epsilon_k r_{nd,k}^* (1 - \hat{\lambda}_{nd,k}\lambda_{nd,k}) + \sum_{d \neq i} r_{nd}^* \zeta_{nd}^*. \tag{28}$$

Using the same logic, we can express the equilibrium conditions specified by Equations 7 and 8 in hat-algebra notation. Solving the updated optimal tariff formula (Equation 22) alongside these equilibrium conditions determines the Nash tariffs and their welfare effects in one simple step. The following proposition outlines this claim.

Proposition 2. *If preferences are described by functional form 4, the Nash tariffs, $\{t_{ni}^*\}_{n \neq i}$, and their effect on wages, $\{\hat{w}_i\}$, and total income, $\{\hat{Y}_i\}$, can be solved as a solution to the following system:*

$$\begin{cases} 1 + t_{ni}^* = 1 + \dfrac{r_{ni}^* + \sum_{j \notin \{i,n\}} t_{ji}^* C_{nj}^{i*}}{D_n^{i*}} & \textit{[optimal tariff]} \\ \hat{\lambda}_{ji,k} = \dfrac{\left[(1 + \hat{t}_{ji,k})\hat{w}_j\right]^{-\epsilon_k}}{\sum_{n=1}^{N} \left(\lambda_{ni,k} \left[(1 + \hat{t}_{ni,k})\hat{w}_n\right]^{-\epsilon_k} \right)}; \quad 1 + \hat{t}_{ji,k} = \dfrac{1 + t_{ji}^*}{1 + \bar{t}_{ji,k}} & \textit{[expenditure shares]} \\ \hat{w}_i w_i \bar{L}_i = \sum_{k=1}^{K} \sum_{j=1}^{N} \left[\dfrac{1}{1 + t_{ij}^*} \hat{\lambda}_{ij,k}\lambda_{ij,k} e_{j,k} \hat{Y}_j Y_j \right] & \textit{[wage bill = sales net of taxes]} \\ \hat{Y}_i Y_i = \hat{w}_i w_i \bar{L}_i + \sum_{k=1}^{K} \sum_{j \neq i} \left[\dfrac{t_{ji}^*}{1 + t_{ji}^*} \hat{\lambda}_{ji,k}\lambda_{ji,k} e_{i,k} \hat{Y}_i Y_i \right] & \textit{[income = wage bill + tax rev.]} \end{cases}$$

where $r_{nd,k}^*$, $q_{nd,k}^*$, $Q_{d,k}^*$, ζ_{nd}^*, C_{nj}^{i*}, and D_n^{i*} are given by Equations 24–28. Importantly, solving the above system requires information on only (i) industry-level trade elasticities, ϵ_k; (ii) applied tariffs, $\bar{t}_{ji,k}$; (iii) observable shares, $\lambda_{ji,k}$ and $e_{i,k}$; and (iv) national income, Y_i.[1]

The system specified by Proposition 2 involves $N(N-1) + 2N$ independent equations and unknowns—namely, $N(N-1)$ Nash tariff rates, $\{t_{ni}^*\}_{n \neq i}$, N wage changes, $\{\hat{w}_i\}$, and

[1]Wage income can be inferred from $\bar{t}_{ji,k}$, $\lambda_{ji,k}$, $e_{i,k}$, and Y_i as $w_i \bar{L}_i = Y_i \left(1 - \sum_{k,j \neq i} \frac{\bar{t}_{ji,k}}{1 + \bar{t}_{ji,k}} \lambda_{ji,k} e_{i,k}\right)$.

N income changes, $\{\hat{Y}_i\}$. Solving this system requires information on a set of observable or estimable sufficient statistics: namely, observable applied tariffs ($\bar{t}_{ji,k}$), expenditure shares ($\lambda_{ji,k}$ and $e_{i,k}$), and national income data (Y_i), which are typically reported in standard datasets, as well as estimated values for industry-level trade elasticities (ϵ_k) that are attainable with standard techniques.

The solution to the system specified by Proposition 2 immediately pins down the prospective cost of a global tariff war for each country i as

$$\%\Delta\text{Real GDP}_i = \hat{Y}_i \Big/ \prod_{k=1}^{K} \left(\hat{\tilde{P}}_{i,k}^{e_{i,k}}\right), \tag{29}$$

where $\hat{\tilde{P}}_{i,k} = \left(\sum_{n=1}^{N} \lambda_{ni,k} \left[(1+\widehat{t}_{ni,k})\hat{w}_n\right]^{-\epsilon_k}\right)^{-1/\epsilon_k}$ denotes the change in destination i–industry k's CES price index. The next section performs these calculations using actual trade and production data from many countries and over many years.

6 Quantitative Implementation

In this section, I employ Proposition 2 to compute the prospective cost of a tariff war for 43 major economies under both the original optimal tariff formula (Equation 11) and the updated optimal tariff formula (Equation 22), and to study how this cost has evolved over time. To solve the system specified by Proposition 2, I need data on the full matrix of industry-level bilateral trade values, $\tilde{X}_{ji,k} \equiv \tilde{P}_{ji,k} Q_{ji,k}$, and applied tariffs, $\bar{t}_{ji,k}$. Knowing these values, I can determine total expenditure, $Y_i = \sum_j \sum_k \tilde{X}_{ji,k}$; wage revenue, $w_i \bar{L}_i = \sum_j \sum_k \tilde{X}_{ij,k}/(1+\bar{t}_{ij,k})$; as well as expenditure shares, $e_{i,k} = \sum_j \tilde{X}_{ji,k}/Y_i$, and $\lambda_{ji,k} = \tilde{X}_{ji,k}/e_{i,k}Y_i$. Below, I describe how the required data is collected from different sources.

Data on Trade Values, Applied Tariffs, and Trade Elasticities. Data on bilateral trade values are taken from the 2016 release of the World Input-Output Database (WIOD, see Timmer et al. (2012)). The dataset spans years 2000 to 2014, covering 43 countries (plus an aggregate of the rest of the world) and 56 industries. The 43 countries featured in the WIOD are listed in the first column of Table 2. Following Costinot and Rodriguez-Clare (2014), I group the industries into 16 industrial categories, assuming that industries belonging to the same category are governed by the same trade elasticity parameter. Importantly, to make the WIOD data compatible with theory, I follow the methodology in

Dekle et al. (2007) to purge the data from underlying trade imbalances. For applied tariffs, I use data from the United Nations Statistical Division, Trade Analysis and Information System (UNCTAD-TRAINS). In line with Caliendo et al. (2015), I assign the simple tariff line average of the effectively applied tariff (AHS) to $\bar{t}_{ji,k}$, and closely follow the methodology outlined in Kucheryavyy et al. (2016) to aggregate the UNCTAD-TRAINS data into individual WIOD industries. Finally, I choose 2014 as the baseline year and employ the triple-difference methodology developed by Caliendo and Parro (2015) to estimate a trade elasticity, ϵ_k, for each of the WIOD industry categories in my analysis.

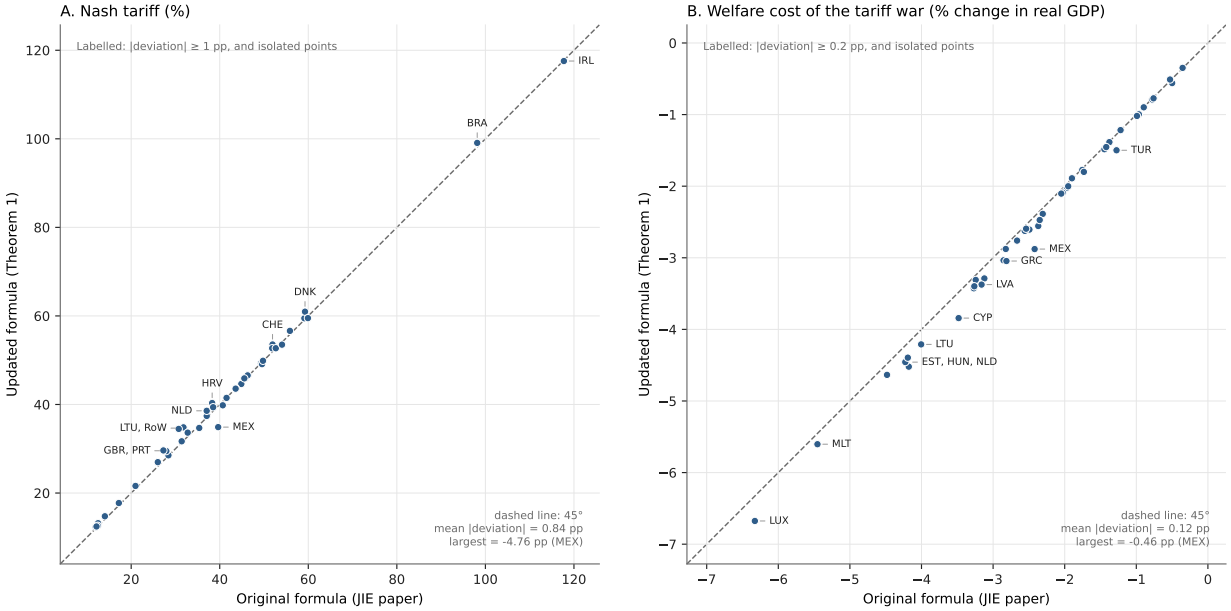
6.1 The Cost of a Global Tariff War for Different Nations

Table 2 reports (i) the computed Nash tariff levels, as well as (ii) the per-cent loss in real GDP as a result of the tariff war for various countries in the baseline Ricardian model under both the original optimal tariff formula (Equation 11) and the updated optimal tariff formula (Equation 22). Recall that in the baseline Ricardian model, tariffs are targeted solely at improving a country's wage relative to the rest of the world. Under the original formula, the Nash tariffs are uniform across exporters and industries and stand at 40.5% for the average economy. Under the updated formula, the Nash tariffs are uniform across industries within each export partner and stand at 41.0% for the average economy (where each country's reported Nash tariff in Table 2 is the simple average across its export partners). In both cases, the heterogeneity in Nash tariffs across countries is driven primarily by the average trade elasticity underlying a country's exports. For instance, the Nash tariffs are significantly lower for Australia (14.1% under the original formula and 14.8% under the updated formula), Norway (17.2% and 17.7%), and Russia (12.2% and 12.5%) who predominantly export primary commodities that are subject to high trade elasticities.

From the perspective of the baseline Ricardian model, the average country loses 2.42% of its real GDP in the event of a tariff war under the original formula, and 2.54% of its real GDP under the updated formula. These losses are driven by pure trade reduction. Even though the losses are quite heterogeneous, all countries lose without exception under both formulas, with smaller countries like Luxembourg (-6.33% under the original formula and -6.68% under the updated formula) and Malta (-5.45% and -5.60%) being the most affected due to their greater reliance on trade and limited market power. By contrast, large economies like China (-0.35% under both formulas), Brazil (-0.50% and -0.56%), Japan (-0.53% and -0.51%), and the United States (-0.76% and -0.77%) experience smaller losses.

Figure 1 compares the Nash tariffs and the welfare losses computed using the original formula (assigned to the x-axis) to those computed using the updated formula. Evidently, the Nash tariffs and the welfare losses predicted by the two formulas are quite similar across countries: the average difference between the two formulas is 0.84 percentage points for Nash tariffs and 0.12 percentage points for the per-cent loss in real GDP, with the largest difference observed for MEX (34.9% under the updated formula compared to 39.7% under the original formula for the Nash tariff, and -2.88% compared to -2.42% for real GDP). For 40 of the 44 economies in the sample, the predicted welfare loss from a global tariff war is slightly higher under the updated formula than under the original formula.

Figure 1: Nash tariffs and welfare losses (year 2014): original vs. updated formula



Note: The source of the data is the 2014 WIOD and UNCTAD-TRAINS. The underlying model is the baseline Ricardian model from Section 2. Each dot corresponds to an individual economy in the 2014 WIOD sample. The x-axis reports the Nash tariffs and the per-cent loss in real GDP computed using the original optimal tariff formula (Equation 11), whereas the y-axis reports the corresponding values computed using the updated optimal tariff formula (Equation 22).

6.2 The Cost of a Global Tariff War Over Time

Building on the computational speed of my methodology, I also compute the cost of a global tariff war in the baseline Ricardian model under both the original formula and the updated formula across many years—from 2000 to 2014 in the case of the WIOD data.

Table 2: The welfare cost of a tariff war (year 2014): original vs. updated formula

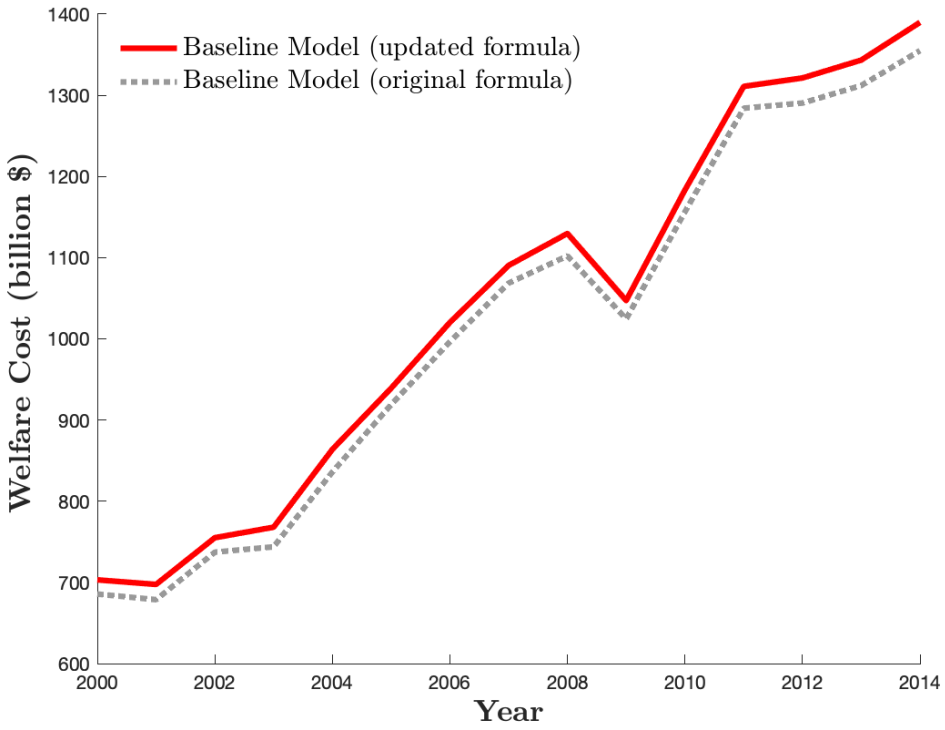
	Baseline Model (original formula)		Baseline Model (updated formula)	
Country	Nash Tariff	%Δ Real GDP	Nash Tariff	%Δ Real GDP
AUS	14.1%	-1.38%	14.8%	-1.38%
AUT	45.7%	-2.82%	45.9%	-2.88%
BEL	55.9%	-3.27%	56.6%	-3.43%
BGR	37.1%	-3.24%	37.4%	-3.31%
BRA	98.2%	-0.50%	99.1%	-0.56%
CAN	21.0%	-2.37%	21.6%	-2.56%
CHE	51.9%	-1.97%	53.6%	-2.02%
CHN	40.7%	-0.35%	39.8%	-0.35%
CYP	12.5%	-3.48%	12.9%	-3.84%
CZE	49.3%	-2.85%	49.3%	-3.04%
DEU	59.1%	-0.96%	59.5%	-1.00%
DNK	59.3%	-2.31%	61.0%	-2.39%
ESP	59.9%	-1.45%	59.5%	-1.48%
EST	28.4%	-4.18%	28.5%	-4.52%
FIN	31.4%	-1.75%	31.7%	-1.77%
FRA	51.8%	-1.73%	52.7%	-1.80%
GBR	27.9%	-2.03%	29.5%	-2.09%
GRC	12.5%	-2.81%	13.2%	-3.05%
HRV	38.3%	-3.12%	40.3%	-3.29%
HUN	52.7%	-4.23%	52.7%	-4.46%
IDN	54.1%	-0.99%	53.5%	-1.02%
IND	49.6%	-0.90%	49.1%	-0.90%
IRL	117.7%	-1.42%	117.6%	-1.45%
ITA	49.8%	-0.78%	49.9%	-0.79%
JPN	44.9%	-0.53%	44.6%	-0.51%
KOR	43.6%	-1.22%	43.7%	-1.22%
LTU	31.8%	-4.00%	34.8%	-4.21%
LUX	12.0%	-6.33%	12.3%	-6.68%
LVA	26.0%	-3.16%	27.0%	-3.37%
MEX	39.7%	-2.42%	34.9%	-2.88%
MLT	12.4%	-5.45%	12.6%	-5.60%
NLD	37.1%	-4.19%	38.6%	-4.40%
NOR	17.2%	-2.05%	17.7%	-2.10%
POL	46.4%	-2.67%	46.6%	-2.76%
PRT	27.3%	-2.49%	29.6%	-2.61%
ROU	32.8%	-2.56%	33.6%	-2.62%
RUS	12.2%	-2.54%	12.5%	-2.59%
SVK	41.5%	-4.48%	41.5%	-4.64%
SVN	46.3%	-3.26%	46.6%	-3.40%
SWE	38.5%	-1.95%	39.4%	-2.00%
TUR	45.6%	-1.28%	45.9%	-1.50%
TWN	35.4%	-2.35%	34.7%	-2.47%
USA	43.6%	-0.76%	43.6%	-0.77%
Average	40.5%	-2.42%	41.0%	-2.54%

Note: The source of the data is the 2014 WIOD and UNCTAD-TRAINS. The underlying model is the baseline Ricardian model from Section 2. Under the original formula, the Nash tariffs and the per-cent loss in real GDP are computed using Equation 11. Under the updated formula, the Nash tariffs (reported as the simple average across export partners) and the per-cent loss in real GDP are computed using Equation 22.

Figure 2 displays the final results. For every year, the cost of a tariff war to the global economy is calculated as the change in real global GDP. To calculate this change, I use yearly data on constant real GDP from the Penn World Tables. I multiply and add the per-cent loss in real GDP for each country by its constant real GDP level in that year.

Based on Figure 2, the prospective cost of a tariff war has multiplied from 2000 to 2014 under both formulas. To provide numbers, under the original formula, the prospective cost has nearly doubled from $685.6 billion in 2000 to $1,355.0 billion in 2014. Under the updated formula, the prospective cost has similarly nearly doubled from $703.1 billion in 2000 to $1,389.8 billion in 2014. In every year between 2000 and 2014, the global welfare cost computed using the updated formula is slightly higher than that computed using the original formula (by $17.5 billion in 2000 and $34.9 billion in 2014), while the overall level and change over time of the prospective cost of a global tariff war remain quite close across the two formulas.

Figure 2: The prospective cost of a tariff war over time



Note: The source of the data is the 2000–2014 WIOD, UNCTAD-TRAINS, and the Penn World Tables. The underlying model is the baseline Ricardian model from Section 2. For every year, the cost of a tariff war to the global economy is calculated as the change in real global GDP (in billion dollars) under the original optimal tariff formula (Equation 11) and the updated optimal tariff formula (Equation 22).

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A Proof of Proposition 1

Step #1: Express Equilibrium Variables and Welfare in Country i. Normalize by wage income in country i, $I_i = w_i \bar{L}_i$, and define

$$m_n = \frac{I_n}{I_i},\ x_n = \ln \frac{w_n}{w_i},\ c_{n,k} = x_n + \ln(1 + t_{ni,k}),\ b_{n,k} = \frac{X_{ni,k}}{I_i},\ b_n = \sum_{k=1}^{K} b_{n,k},\ B_k = \sum_{n=1}^{N} b_{n,k}, \tag{30}$$

with $x_i = c_{i,k} = 0$ and $m_i = 1$. Expenditure shares in country i depend only on $c = \{c_{n,k}\}_{n \neq i,k}$, whereas expenditure shares and q-shares in foreign destinations $d \neq i$ depend only on $x = \{x_n\}_{n \neq i}$ because applied tariffs in the rest of the world are held fixed. Conversely, any (c, x) pins down unique tariffs $1 + t_{ni,k} = \exp(c_{n,k} - x_n) > 0$. By Equation 14,

$$b_{n,k} = \frac{e_{i,k} \lambda_{ni,k}}{\rho_i (1 + t_{ni,k})}, \qquad \sum_{n=1}^{N} b_n = \sum_{k=1}^{K} B_k = 1. \tag{31}$$

The labor market clearing conditions in the rest of the world ($j \neq i$) and social welfare in country i (aside from the constant $\ln \bar{L}_i$) can be stated as

$$H_j(c, x) \equiv m_j - b_j - \sum_{d \neq i} m_d q_{jd}(x) = 0, \qquad V(c, x) = -\ln \rho_i - \sum_{k=1}^{K} e_{i,k} \ln(\tilde{P}_{i,k} / w_i). \tag{32}$$

By Walras' law, adding $H_j(c, x) = 0$ over $j \neq i$ implies the labor market clearing condition in country i.

Step #2: Partial Derivatives with Respect to Tariffs and Non-Singularity. For a foreign origin $a \neq i$, taking partial derivatives with respect to $c_{a,k}$ holding x fixed yields

$$\partial_{c_{a,k}} \lambda_{ni,g} = -\mathbf{1}\{g = k\} \epsilon_k \lambda_{ni,k} (\mathbf{1}\{n = a\} - \lambda_{ai,k}),\ \ \partial_{c_{a,k}} \ln \rho_i = \epsilon_k \lambda_{ai,k} B_k - (1 + \epsilon_k) b_{a,k}, \tag{33}$$

$$V_{c_{a,k}} = (1 + \epsilon_k) b_{a,k} - \lambda_{ai,k} (e_{i,k} + \epsilon_k B_k),$$

$$H_{j,c_{a,k}} = (1 + \epsilon_k)(\mathbf{1}\{j = a\} b_{j,k} - b_j b_{a,k}) - \epsilon_k \lambda_{ai,k} (b_{j,k} - b_j B_k). \tag{34}$$

To verify that the matrix $(H_{j,c_{a,k}})_{j,a \neq i}$ is non-singular for each industry k, suppose its product with a vector $v = (v_a)_{a \neq i}$ equals zero. Define $u_j = b_{j,k} / b_j > 0$, $s = \sum_{a \neq i} b_{a,k} v_a$, $\ell = \sum_{a \neq i} \lambda_{ai,k} v_a$, and $\gamma = (1 + \epsilon_k) s - \epsilon_k B_k \ell$. Rearranging row j of $\sum_{a \neq i} H_{j,c_{a,k}} v_a = 0$ yields $v_j = \frac{\gamma}{(1+\epsilon_k) u_j} + \frac{\epsilon_k \ell}{1+\epsilon_k}$. When we multiply v_j by $b_{j,k}$ and sum over $j \neq i$, we get $\gamma b_i + \epsilon_k b_{i,k} \ell = 0$. When we instead multiply by $\lambda_{ji,k}$ and sum over $j \neq i$, we get

$\ell = \frac{\gamma \sum_{j \neq i} \lambda_{ji,k}/u_j}{1+\epsilon_k \lambda_{ii,k}}$. Plugging ℓ into the first identity implies $\gamma = 0$ because $b_i > 0$ and all terms are positive. Accordingly, $\ell = s = 0$ and $v_j = 0$ for all $j \neq i$, which establishes that $(H_{j,c_{a,k}})_{j,a \neq i}$ is non-singular.

Step #3: Uniformity Across Industries. Form the Lagrangian with multipliers μ_j, $\mathcal{L} = V(c,x) + \sum_{j \neq i} \mu_j H_j(c,x)$, and define

$$\mu_i = 0, \qquad M = \sum_{j \neq i} \mu_j b_j, \qquad v_n = 1 + \mu_n - M, \qquad S_k = \sum_{n=1}^{N} v_n b_{n,k}. \tag{35}$$

Adding over k and n gives $\sum_{k=1}^{K} S_k = \sum_{n=1}^{N} v_n b_n = 1$. By Equation 34, the first-order condition with respect to $c_{a,k}$ ($a \neq i$) and its sum over $a \neq i$ within industry k are

$$(1+\epsilon_k) b_{a,k} v_a = \lambda_{ai,k}(e_{i,k} + \epsilon_k S_k), \ (1+\epsilon_k \lambda_{ii,k}) S_k = e_{i,k}(1-\lambda_{ii,k}) + (1+\epsilon_k)(1-M) b_{i,k}. \tag{36}$$

Let $z = (1-M)/\rho_i$, plug $b_{i,k} = e_{i,k}\lambda_{ii,k}/\rho_i$ into the second equation in 36, and sum over $k = 1, \ldots, K$. Using $\sum_{k=1}^{K} S_k = \sum_{k=1}^{K} e_{i,k} = 1$, we get $0 = (z-1) \sum_{k=1}^{K} \frac{e_{i,k}(1+\epsilon_k)\lambda_{ii,k}}{1+\epsilon_k \lambda_{ii,k}}$. Since the sum is strictly positive, $z = 1$, which implies $M = 1 - \rho_i$ and $S_k = e_{i,k}$ for every industry k. Equation 36 then reduces to $b_{a,k}(\rho_i + \mu_a) = e_{i,k}\lambda_{ai,k}$. Combining this with Equation 31 establishes that country i's optimal tariff on exporter $a \neq i$ is uniform across industries:

$$1 + t^*_{ai,k} = 1 + \frac{\mu_a}{\rho_i} \equiv 1 + t^*_{ai}, \qquad \mu_a = \rho_i t^*_{ai}. \tag{37}$$

Step #4: General Equilibrium Income Effects in the Rest of the World. Holding applied tariffs $\bar{t}_{jd,k}$ in every foreign destination $d \neq i$ fixed, taking the partial derivative of $\lambda_{jd,k}$ with respect to x_a ($a \neq i$) gives $\partial_{x_a} \lambda_{jd,k} = -\epsilon_k \lambda_{jd,k}(\mathbf{1}\{j = a\} - \lambda_{ad,k})$. Since $\frac{e_{d,k}\lambda_{jd,k}}{(1+\bar{t}_{jd,k})\rho_d} = q_{jd,k}$, taking the partial derivative of ρ_d in Equation 14 with respect to x_a yields

$$\partial_{x_a} \ln \rho_d = \sum_{k=1}^{K} \epsilon_k \left[\lambda_{ad,k} \sum_{j=1}^{N} q_{jd,k} - q_{ad,k} \right] = \zeta_{ad}, \qquad \partial_{x_a} \ln Y_d = \mathbf{1}\{a = d\} - \zeta_{ad}. \tag{38}$$

Equation 38 accounts for the general equilibrium income effect in foreign destination $d \neq i$ generated by tariff revenue adjustments. Correspondingly, the partial derivative of $q_{jd,k}$ with respect to x_a is

$$\partial_{x_a} q_{jd,k} = q_{jd,k} \left[-\epsilon_k \mathbf{1}\{j = a\} + \epsilon_k \lambda_{ad,k} - \zeta_{ad} \right]. \tag{39}$$

Step #5: First-Order Conditions with Respect to Wages and Characterizing t^*_{ni}. Holding c fixed, taking partial derivatives of Equations 31–32 with respect to x_a ($a \neq i$) gives $V_{x_a} = -b_a$, $\partial_{x_a} b_j = b_j(\mathbf{1}\{j = a\} - b_a)$, $\partial_{x_a} m_j = \mathbf{1}\{j = a\} m_j$, and, using Equation 39 and $m_j - b_j = \sum_{d \neq i} m_d q_{jd}$,

$$\begin{aligned} H_{j,x_a} = \mathbf{1}\{j = a\} \sum_{d \neq i} \sum_{k=1}^{K} m_d (1 + \epsilon_k) q_{jd,k} + b_j b_a - m_a q_{ja} \\ - \sum_{d \neq i} \sum_{k=1}^{K} m_d \epsilon_k q_{jd,k} \lambda_{ad,k} + \sum_{d \neq i} m_d q_{jd} \zeta_{ad}. \end{aligned} \tag{40}$$

Using $\mu_j = \rho_i t^*_{ji}$, the first-order condition with respect to x_a is $-b_a + \rho_i \sum_{j \neq i} t^*_{ji} H_{j,x_a} = 0$. By Equation 14 and industry uniformity (Equation 37), country i's tariff revenue identity implies $\sum_{j \neq i} t^*_{ji} b_j = (1 - \rho_i)/\rho_i$. Plugging Equation 40 into the first-order condition, noting that $b_a / m_a = r_{ai}$ and $m_d q_{jd,k} / m_a = (\omega_j / \omega_a) r_{jd,k}$, and setting $A_a \equiv \sum_{d \neq i} \sum_{k=1}^{K} (1 + \epsilon_k) r_{ad,k}$ and $D_a \equiv A_a - C_{aa}$ delivers

$$A_a t^*_{ai} = r^*_{ai} + \sum_{j \neq i} C^*_{aj} t^*_{ji}, \qquad D^*_a t^*_{ai} = r^*_{ai} + \sum_{j \notin \{i,a\}} C^*_{aj} t^*_{ji}. \tag{41}$$

Here $A_a - C_{aa}$ equals D_a in Equation 20. To verify that $D_a > 0$, note that for each foreign destination $d \neq i$, using $r_{ad,k} = (\omega_d / \omega_a) q_{ad,k}$ and Equation 18 gives

$$\begin{aligned} &\sum_{k=1}^{K} \epsilon_k r_{ad,k} (1 - \lambda_{ad,k}) + r_{ad} \zeta_{ad} \\ &\quad = \frac{\omega_d}{\omega_a} \sum_{k=1}^{K} \epsilon_k \left[(1 - q_{ad})(1 - \lambda_{ad,k}) q_{ad,k} + q_{ad} \lambda_{ad,k} (Q_{d,k} - q_{ad,k}) \right] > 0. \end{aligned}$$

Adding $\sum_{d \notin \{i,a\}} r_{ad} \geq 0$ establishes $D_a > 0$. Finally, uniformity across industries implies $r_{ji} = \frac{\lambda_{ji} Y_i}{(1 + t_{ji}) I_j}$, so that $t^*_{ji} = \frac{\lambda^*_{ji} y^*_i}{\rho^*_j r^*_{ji} y^*_j} - 1$. Plugging this expression for $j \notin \{i, a\}$ into Equation 41 and rearranging yields Equation 22. □